

Coastal Engineering Technical Note



SPECTRAL WAVE MODELLING TECHNOLOGY

<u>Introduction</u>: Over the past 40 years, spectral wave modelling has made significant advances, from the theoretical framework of Phillips (1957) and Miles (1957), describing why waves develop and grow, through the era of hand calculations and nomograms where the engineer was able to estimate wave heights and periods for design projects, to the sophistication of spectral wave models of today that take advantage of our advanced *SUPER-COMPUTER* facilities. In the last 10 years, there have been numerous publications describing spectral models and their respective *generation* level, i.e. first, second, and more recently, third generation. To the first time user, this can be very confusing and cumbersome. The intent of this technical note is to define, summarize, and identify the spectral wave modelling capabilities that exist today.

Background: The evolution of spectral wave modelling began in the early 1940's with forecasting waves for planning and execution of military operations during World War II. Sverdrup and Munk (1947) used physical arguments to approximate a generalized version of wind-wave growth. From a statistical basis related to the sea-state, the significant wave method was formed. Further theoretical work, using the energy balance equation and non-dimensional analysis, resulted in defining two wave parameters: the significant wave height H_a and significant period T_a. Sverdrup and Munk determined that H_a and T_a could be directly related to the wind speed, a fetch length or duration. These curves and a parallel study, Bretschnider (1952), provided the foundation for early wave forecasting methods. These methods were further refined into a spectral approach by Pierson, Neuman, and James (1955), where a series of nomograms describing a frequency spectrum could be constructed from various wind speed, fetch lengths and durations. These techniques, which were strongly founded on empirical results, remained a basis for coastal design for nearly 20 years.

<u>First-Generation Modelling</u>: In the early 1960's, there were numerous groups developing what we would classify as *I*st generation wave models (1G). The governing equation used for deep-water application is the energy balance or transport equation, and for deep-water application is:

$$\frac{\partial E}{\partial t} + \vec{c_g} \cdot \vec{\nabla} E = \sum S_i$$

where E is the two-dimensional wave spectrum. The value of E depends on the spatial coordinates x and y, the temporal variable t, the frequency domain f, and the direction domain θ . The parameter \vec{c}_g is the group velocity and is governed by x,y,f, and θ . The parameter S_i defines source/sink terms, such as the atmospheric input S_{in} , the nonlinear wave-wave interaction S_{nl} , and the high frequency dissipation S_{ds} . The goal is to solve for the time rate of change in E, or the directional spectra in a prescribed gridded system.

Quantification of the source/sink terms was initiated from theoretical considerations by Miles (1957) and Phillips (1957), termed as the *Miles-Phillips Mechanisms* for wind-wave growth. An energy source term transfers energy into the system, while an energy sink term removes energy from the system. In theory, the sum of all source/sink terms is balanced, or equal to zero. Miles' and Phillips' presumptions were that momentum was transferred from the winds to the free surface via an atmospheric input expression through two terms: an external turbulent pressure forcing mechanism, and a linear

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Over the past 40 years, spectral wave modelling has made significant advances, from the theoretical framework of Phillips (1957) and Miles (1957), describing why waves develop and grow,through the era of hand calculations and nomograms where the engineer was able to estimate wave heights and periods for design projects, to the sophistication of spectral wave models of today that take advantage of our advanced SUPER-COMPUTER facilities. In the last 10 years, there have been numerous publications describing spectral models and their respective generation level, i.e. frost, second, and more recently, third generation. To the first time user, this can be very confusing and cumbersome. The intent of this technical note is to define, summarize, and identify the spectral wave modelling capabilities that exist today.					
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feedback mechanism, mathematically represented by:

$$S_{in} = A + B \cdot E$$

The A term (Phillips' mechanism) initiates linear growth with respect to time; and the BE term (Miles' instability mechanism) sustains an exponential growth over time. Based on this equation, windwave growth would never stop. However, additional work by Phillips (1958) on the universal equilibrium range formed the foundation to control wind-wave growth. In the final growth stages, there is a weak coupling between the input of energy from the winds and the limit in relative steepness via the dissipation term. Later, Pierson and Moskowitz (1962) formulated an assumed limit to the frequency spectrum described as fully developed conditions.

The Spectral Ocean Wave Model (Pierson 1982) and a more recent version Global Spectral Ocean Wave Model (Clancy et al. 1986), used by the Fleet Numerical Meteorology and Oceanography Center (FNMOC, Monterey, CA), and also the Ocean Data Gathering Program model (Cardone et al. 1976) are examples of first- generation spectral wave models. For accurate representations of wind-wave growth in model simulations, the A term had to be set several orders of magnitude greater than indicated in the turbulent pressure measurements. In addition to this, the B term had to be increased by an order of magnitude beyond theoretical limits set by Miles (1957), as derived from work by Jeffreys (1924). Field measurements by Barnett and Wilkerson (1967) observed a phenomenon they termed overshoot in the frequency spectrum (i.e. above the equilibrium range, Figure 1) during wind-wave growth. Spectrally derived 1G models could not reproduce this property with only the Miles-Phillips mechanism. Although these models were simplistic, they have for many years produced reasonable results (Wittmann and Clancy 1991). The reasoning is quite simple. It comes from improvements in the estimates of the wind field description, and also years of tuning the respective wave model A and B terms.

Second-Generation Modelling: While 1st generation wave modelling was being carried out, a major theoretical breakthrough was surfacing. Wind-wave growth based on the Miles-Phillips mechanisms assumed that physical processes were based on a coupled linear system between the atmospheric input and a dissipation term. It appeared that a coupled nonlinear process could explain transfers of energy between frequency bands, and the eventual migration of the spectral peak toward lower frequencies. Hasselmann (1962, 1963a,b) introduced the concept of the nonlinear wave-wave interaction that seemed to define these processes rather well. Solution of the Boltzmann integrals (see Hasselmann (1962)) produced estimates of the nonlinear wave-wave interaction term. This contribution provided the foundation for second-generation wave modelling. Many theoreticians debated the strength of S_n in the evolution of ocean wave spectra. It was not until careful laboratory experiments (e.g. Mitsuyasu 1968), and field experiments (JOint North Sea Wave Project or JONSWAP (Hasselmann et al. 1973)) measuring winds and wave spectra that the effect of S_{n1} was generally accepted. From these experiments, new ideas of wave physics were developed, as well as formulations for fetch and duration growth rate expressions. More importantly, these measurements supported theoretical findings that the nonlinear wave-wave interaction mechanism had a significant impact in the physics of ocean wave spectra. The atmospheric input would transmit energy into the system, while the wave-wave interaction mechanism was responsible for the exchange of energy between frequency bands. This allowed for the migration of energy into the forward face (low frequencies) and high-frequency rear face of the spectrum (Figure 2). Coupling the nonlinear wave-wave interaction term and the atmospheric source input selectively reduced the contribution of the winds by an order of magnitude, approaching the results envisioned by Miles and Phillips.

The spectral shape, based on historical (Barnett and Wilkerson 1967), and more recently

JONSWAP data (Hasselmann et al. 1973) was found to be strongly influenced by the S_{n1} term. The overshoot-undershoot phenomenon was also characteristic of the generalized shape of S_{nl} As indicated in the JONSWAP (Figure 2). experiment (Hasselmann et al. 1973), and its subsequent spectral shape, the peak in the spectral energy was nearly a factor of 3 greater than in fully developed conditions, a consequence of S_{nl} . Hence, on theoretical and also experimental grounds, wind-wave growth seemed to be well understood. The spectral energy balance was now linked to a nonlinear system with three terms (S_{in} , S_{nl} , and S_{ds}) rather than two decoupled, linear processes (S_{in} and S_{dn}) described in 1G models. Barnett (1968) and Ewing (1971) were credited with the development of initial spectral wave models containing the S_{nl} term. However, the relative strength of the nonlinear wave-wave interaction source term was very weak, and growth was essentially controlled by the S_{in} and S_{ds} terms.

Second-Generation Modelling: During the late 1960's and through the 1980's, there was a mass proliferation of second generation wave models. ADWAVE, the original Wave Information Studies spectral hindcast model (Resio 1981) and SHALWV (Hughes and Jensen 1984), are examples of these models historically, while WISWAVE (Hubertz 1992) is an example of the model presently used in the Corps of Engineers. There are many other 2G spectral models used throughout the world, and examples are summarized in The SWAMP Group (1985) and Khandekar (1989).

These 2G models, although different in structure or numerical solution method, contain the S_{in} , S_{nl} , and S_{ds} relationships in one form or another. Terms for both the atmospheric input (e.g. Snyder et al. 1981) and the dissipation sink (e.g. Komen et al. 1984) are well-posed, and can be applied to a discrete frequency direction space in a spectral wave model. The greatest problem to overcome in these models was how to evaluate the nonlinear wave-wave interaction term. The exact solution of the three-dimensional Boltzmann

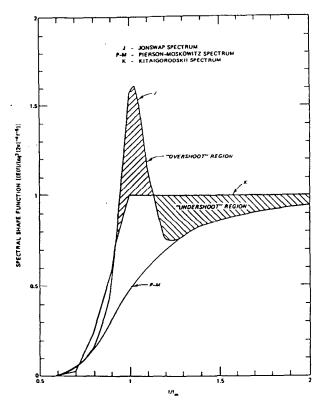


Figure 1. Schematic of Overshoot and Undershoot (Resio 1981).

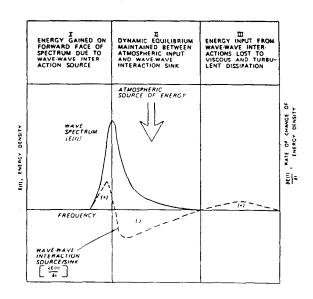


Figure 2. Schematic of the Nonlinear Wave-Wave Interaction Term.

integral is computationally restrictive. Parameterizations of S_{nl} were also found to be restricted to very narrow banded spectra, like that proposed by Resio (1981). A database containing S_{nl} estimates for a much larger class of spectra (e.g. Hasselmann and Hasselmann 1981) would be nearly impossible to generate as indicated by Resio (1993). To mix integral parameterizations (i.e. total energy flux out of the rear face toward the forward face resulting from S_{nl}) with a discrete approximation to the S_{in} and S_{ds} could be considered technically incorrect. Thus, the only viable alternative solution method available is to formulate all source/sink terms as the net energy transfer. This is what is done in WISWAVE (Hubertz 1992).

The number of degrees of freedom between the description of the source/sink terms and the actual spectrum becomes a significant deficiency to overcome. Modelling the source/sink terms is carried out in the form of total energy (or momentum flux) into the system, and subsequent transfer and dissipation as illustrated in Figure 2. The energy must then be redistributed into the discrete frequency/direction domain describing the spectrum. This can lead to the development of unrealistic spectral distributions that can only be controlled by restraining the spectrum to some predetermined level. Declaring an a priori limit to the frequency spectrum is the greatest deficiency in 2G modelling (The SWAMP Group 1985; Resio 1993). Selection of an appropriate limiting form of the spectrum has a certain degree of uncertainty. The slope of the rear face of a spectrum was initially defined by Phillips (1958) as f⁻⁵, and supported by the JONSWAP data (Hasselmann et al. 1973). Toba (1973) found from his measurements that the slope of the rear face in spectra followed an f⁻⁴ shape, and was supported on theoretical grounds by Kitiagordskii (1983), and also Resio and Perrie (1989). Secondly, the construction of this spectral form requires as little as two arguments (the peak frequency, f_m, and a form of the Phillips' equilibrium constant, α), to as many as five in a JONSWAP-type spectrum (Hasselmann et al 1973). Scaling of these parameters has not been shown to be consistent. From self-similarity principles (Kitiagordskii 1983), a given spectral shape is not completely in error, as long as the wave field is at or nearly at equilibrium conditions. Scaling of the energy levels below equilibrium conditions, or during complex meteorological situations as in rapidly changing wind fields (e.g. frontal passages, hurricanes) or mixed wind-sea/swell conditions may sufficiently diminish the accuracy level of the spectrum. Uncertainties in reconstructing a frequency/direction spectrum would, in turn, be reflected in the mean wave parameters of, height, period, and direction.

This is not to say WISWAVE, or any other 2G spectral wave model provides inferior results; on the contrary, they have been shown to yield very realistic wave estimates for a wide variety of meteorological conditions (e.g. Hubertz et al. 1991). Like their predecessors, 2G models improve with age. As these models are exercised for varieties of storm scenarios, improvements can be made to the parametric formulations, leading to improved spectral results.

Third-Generation Modelling: One of the overriding conclusions by the SWAMP Group was the vast differences between individual 2G wave models for simple, academic tests performed as a first-order validation study. As illustrated in Figure 3, comparisons are made between nine wave models, and WISWAVE results for fetch-limited wave growth. Model results range by as much as a factor of 4 with respect to nondimensional wave energy. These results were very surprising considering the simple nature of the test, and the presumption that all 2G spectral wave models should produce very similar results. The diversity in these results was thought to be attributed to the active wind parameter specification embedded in wave growth formulations. Whether or not this was the case, these results spawned a concentrated effort focusing on the next class of ocean spectral wave modelling.

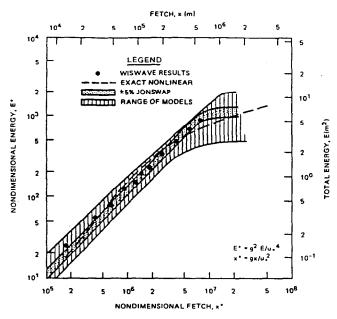


Figure 3. Growth Rate of Nondimensional Total Energy versus Nondimensional Fetch.

As the SWAMP Group convened, a new focus toward spectral modelling technology became evident. Two distinct recommendations were cited, the first was that none of the present (being the mid-1980's) 2G models were applicable for all wind conditions, and secondly, the restriction on the spectral shape severely hampered the final results. The European wave model community coordinated their efforts and embarked on the development of a third-generation spectral model. The work was carried out within the framework of the WAM (Wave Modelling) program (see WAMDI Group (1988)). In the following years, a wave model called 3GWAM (or sometimes referred to as WAM) was developed. Four other 3G models have been developed by various individuals that incorporate the 3G concepts. They are WAVEWATCH (Tolman 1989), Nedwam (Burgers 1990), OWI-3G (Khandekar et al. 1994), and the Full Boltzmann Model developed under contract though the Coastal Research and Development Program (Resio and Perrie 1989; Resio and Perrie 1991; Resio 1993). Differences in these 3G models are generally found in the evaluation of the source/sink term expressions, and predominantly S_{nl}. The actual definition of a 3G model is a rather tenuous one and seems to vary from author to author. In general, 3G models have the following characteristics:

- Description and solution of the source/sink mechanisms are based on first principles, defined discretely in the frequency/direction domain, and not formulated in a parametric or empirical framework.
- 2. There is no a priori limit on the spectral shape. The resulting spectrum is defined from the balance in the source/sink term specification.
- The nonlinear wave-wave interaction term is solved explicitly, and is consistent with the number of degrees of freedom contained in the description of the frequency/directional spectrum.

One has to realize that although technically 3G models are better posed in the evaluation of the physical processes, the physical processes dictated by equational formulations still may not be sufficiently adequate. Recent evidence from the Surface Wave Dynamics Experiment show that 3GWAM has produced extremely precise mean wave parameter estimates and frequency and frequency/directional estimates when compared to measurements (Graber et al. 1991; Jensen et al. 1991; Cardone et al. 1994). In an operational mode at the European Centre for Medium Range Weather Forecasts, 3GWAM has, on a daily basis, provided very accurate wave forecasts (Zambresky 1989, Günther et al. 1993, Komen and Hasselmann 1994). The FNMOC has been successfully beta testing 3GWAM over the last 3 years and will replace GSWOM during 1994 (Wittmann 1992). The Naval Oceanographic Office is also implementing 3GWAM for their wave forecasting and the estimates are consistent with measurements (Farrar and Johnson 1992). In general, there is a direction among U.S. government agencies and worldwide forecasting and research centers toward the use of 3G modelling techniques.

<u>Summary</u>: Spectral ocean wave modelling has made great strides over the last 40 years, from initial 1G through 3G models. All three classes have their respective use in the estimation of deep-water wave conditions. One has to realize that any particular choice in modelling class gives rise to a priori assumptions governing the modelling technique. It does appear that 3G spectral modelling is a viable alternative for future wave-related work in the Corps of Engineers. These models better approximate the physical processes that affect the sea states than their predecessors. Despite this, no model is perfect, and research efforts in the growth, propagation, and transformation of spectral wave processes will continue for a long time.

Additional Information: Contact Dr. Robert E. Jensen, CEWES-CR, (601-634-2101) directly.

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